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DYNAMIC D-Q AXIS MODELING OF THREE PHASE INDUCTION MOTOR IN DIFFERENT REFERENCE FRAMES

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ABSTRACT

This paper presents the dynamic d-q axis modeling of three phase squirrel cage induction motor in different reference frames i.e. stationary, rotor, and synchronous rotating reference frames and observed the dynamic response of the induction motor using MATLAB/SIMULINK software environment. The d-q reference frame theory for induction motor helps to design process of motor-drive systems eliminating design mistakes and in the prototype construction and testing. We observed that the machine outputs such as electromagnetic torque, speed, actual stator phase currents and magnitude of stator flux linkages are all the same regardless of reference frames. We also observed that line start produces higher currents, flux linkages and severe torque pulsations also.

KEYWORDS: Dynamic Model, MATLAB/SIMULINK, Induction Motor, Reference Model, dqo, Flux Linkages

INTRODUCTION

The three-phase induction motors are the most widely used electric motors in industry. The 3-phase induction motors are simple, rugged, low-priced, self –starting capability easy to maintain and can be manufactured with characteristics to suit most industrial requirements. At the time of starting the induction machine draws higher currents produces voltage oscillations and torque pulsations causes power quality problems in power system network. In order to observe such problems the dynamic d-q model is necessary. Besides that high performance drive control such as vector and field oriented control is based on the dynamic d-q model of the machine. Reference frames are nothing but observer platform gives a unique view of the system. The dynamic behaviour of the machine can be observed in following reference frames i.e. 1) Stationary (Stator) reference frame 2) Rotor reference frame 3) Synchronously rotating reference frame. Instead of deriving the transformations for each and every particular reference frame, it is advantageous to derive the general transformation for an arbitrary rotating reference frame. The analysis of reference frame model can be derived by substituting the appropriate frame speed and position. Selection of particular reference frame dependant on the type of applications of induction motor drive.

DYNAMIC D-Q MODEL

The dynamic D-Q machine model a three phase machine can be represented by an equivalent two-phase machine. Where $d^s - q^s$ correspond to stator direct and quadrature axes, and $d^r - q^r$ correspond to rotor direct and quadrature axes. Although it is somewhat simple, the problem of the time varying parameters still remains. R. H. Park, in the 1920s, proposed a new theory of electric machine analysis to solve this problem. He formulated a change of variables which, in effect, replaced the variables associated with the stator windings of a synchronous machines with variables

associated with fictitious windings rotating with the rotor at synchronous speed. Due to that transformation time varying inductances can be eliminated.

THREE PHASE TO TWO PHASE TRANSFORMATION

In order to reduce this complexity the transformation of axes from 3 – Φ to 2 – Φ is necessary. A dynamic model for the three phase induction machine if the equivalence between three and two phases is established. During the transformation mmf produced in the three phase is equal to the mmf produced in two phase.

In the three phase induction machine transform the three phase stationary reference frames $(a - b - c)$ variables into two-phase stationary reference frames $(d^s - q^s)$ variables and transform these to synchronous rotating reference frame $(d^e - q^e)$, and vice versa. Assume that the $d^s - q^s$ axes are oriented at θ angle, the voltages are been resolved into the following components

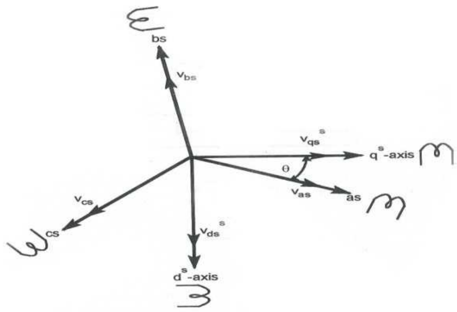


Figure 1: 3-ϕ to 2-ϕ Transformation

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos(\theta - 120^0) & \sin(\theta - 120^0) & 1 \\ \cos(\theta + 120^0) & \sin(\theta + 120^0) & 1 \end{bmatrix} \begin{bmatrix} v_{qs}^s \\ v_{ds}^s \\ v_{os}^s \end{bmatrix} \tag{1}$$

The corresponding inverse relation is

$$\begin{bmatrix} v_{qs}^s \\ v_{ds}^s \\ v_{os}^s \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - 120^0) & \cos(\theta + 120^0) \\ \sin\theta & \sin(\theta - 120^0) & \sin(\theta + 120^0) \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} \tag{2}$$

For Stationary Reference Frame θ=0

Rotor Reference Frame $\theta = \theta_r$

Synchronously Rotating Reference Frame $\theta = \theta_e$

MATHEMATICAL MODELING IN ARBITARY REFERENCE FRAME

A three phase induction motor can be modeled using dq0 reference frame theory. Assuming windings are having number of turns on both of the reference frames,

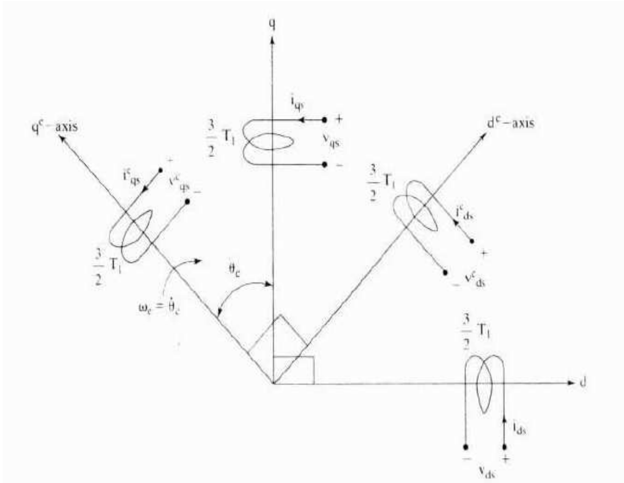


Figure 2: Stationary to Arbitrary Reference Frame

The induction motor model in arbitrary reference frames [1] is obtained. It is given below

$$\begin{bmatrix} v_{qs}^c \\ v_{ds}^c \\ v_{qr}^c \\ v_{dr}^c \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_c L_s & L_m p & \omega_c L_m \\ -\omega_c L_s & R_s + L_s p & -\omega_c L_m & L_m p \\ L_m p & (\omega_c - \omega_r) L_m & R_r + L_r p & (\omega_c - \omega_r) L_r \\ -(\omega_c - \omega_r) L_m & L_m p & -(\omega_c - \omega_r) L_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qs}^c \\ i_{ds}^c \\ i_{qr}^c \\ i_{dr}^c \end{bmatrix} \quad (3)$$

Where

$$\omega_r = \dot{\theta}_r$$

The stator and rotor flux linkages in the arbitrary reference frames [1] are defined as

$$\begin{aligned} \lambda_{qs}^c &= L_s i_{qs}^c + L_m i_{qr}^c \\ \lambda_{ds}^c &= L_s i_{ds}^c + L_m i_{dr}^c \\ \lambda_{qr}^c &= L_r i_{qr}^c + L_m i_{qs}^c \\ \lambda_{dr}^c &= L_r i_{dr}^c + L_m i_{ds}^c \end{aligned} \quad (4\text{-}7)$$

Where L_s is the inductance of Stator

Lr is the inductance of the Rotor

Lm is the mutual inductance between stator and rotor

The voltage Equations in flux linkages [1] are

$$V_{qs}^c = R_s i_{qs}^c + \omega_c \lambda_{ds}^c + p\lambda_{qs}^c \quad (8)$$

$$v_{ds}^c = R_s i_{ds}^c - \omega_c \lambda_{qs}^c + p\lambda_{ds}^c \quad (9)$$

$$v_{dr}^{c} = R_r i_{dr}^{c} - (\omega_c - \omega_r)\lambda_{qr}^{c} + p\lambda_{dr}^{c} \tag{10}$$

$$v_{qr}^{c} = R_r i_{qr}^{c} + (\omega_c - \omega_r)\lambda_{dr}^{c} + p\lambda_{qr}^{c} \tag{11}$$

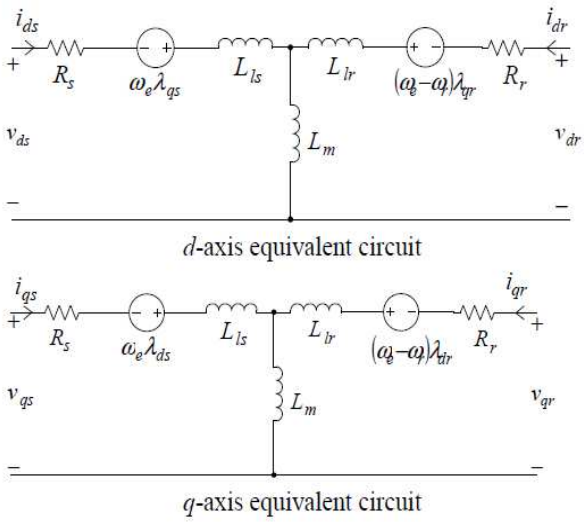


Figure 3: d-q dynamic Equivalent Circuit for an Induction Motor

The currents in flux linkages [1] are

$$i_{qs}^{c} = \frac{L_r \lambda_{qs}^{c} - L_m \lambda_{qr}^{c}}{\Delta_1} \tag{12}$$

$$i_{ds}^{c} = \frac{L_r \lambda_{ds}^{c} - L_m \lambda_{dr}^{c}}{\Delta_1} \tag{13}$$

$$i_{qr}^{c} = \frac{L_s \lambda_{qr}^{c} - L_m \lambda_{qs}^{c}}{\Delta_1} \tag{14}$$

$$i_{dr}^{c} = \frac{L_s \lambda_{dr}^{c} - L_m \lambda_{ds}^{c}}{\Delta_1} \tag{15}$$

Where $\Delta_1 = L_s L_r - L_m{}^2$

The electromagnetic torque in stator flux linkages and stator currents

$$T_e = \frac{3}{2}\frac{P}{2}(i_{qs}^{c}\lambda_{ds}^{c} - i_{ds}^{c}\lambda_{qs}^{c}) \tag{16}$$

For Stationary Reference Frame $\omega_c = 0$

Rotor Reference Frame $\omega_c = \omega_r$

Synchronously RotatingReference Frame $\omega_c = \omega_e$

The equation of motion of the motor is found using the well-known formula for the conservation of angular momentum

$$J \frac{d\omega}{dt} = T_e - T_l \tag{17}$$

Where ω is the mechanical angular velocity of the rotor. The moment of inertia J is the sum of the moments of inertia of the external load and the rotor. The external load torque $T_l$is a function of the angular velocity ω and includes any dampingterms, such as that due to bearing friction.

The electric angular velocity ω_r is related to the mechanical angular velocity through

$$\omega_r = \frac{p}{2}\omega \tag{18}$$

MATLAB/SIMULINK MODELING OF INDUCTION MOTOR IN ALL REFERENCE FRAMES

An induction motor [1] has the following constants and ratings:

200v, 4pole, 3phase, 60Hz, Connected $R_s = 0.183\Omega$, $R_r = 0.277\Omega$, $L_m = 0.0538H$, $L_s = 0.0553H$, $L_r = 0.056$H

B=0, Load torque=$T_1 = 0$ N.m, J=0.0165kg-m^2, base power is 5 HP.

The motor is standstill. A set of balanced three-phase voltages at 70.7% of rated values at 60Hz is applied.

STATIONARY REFERENCE FRAME MODEL

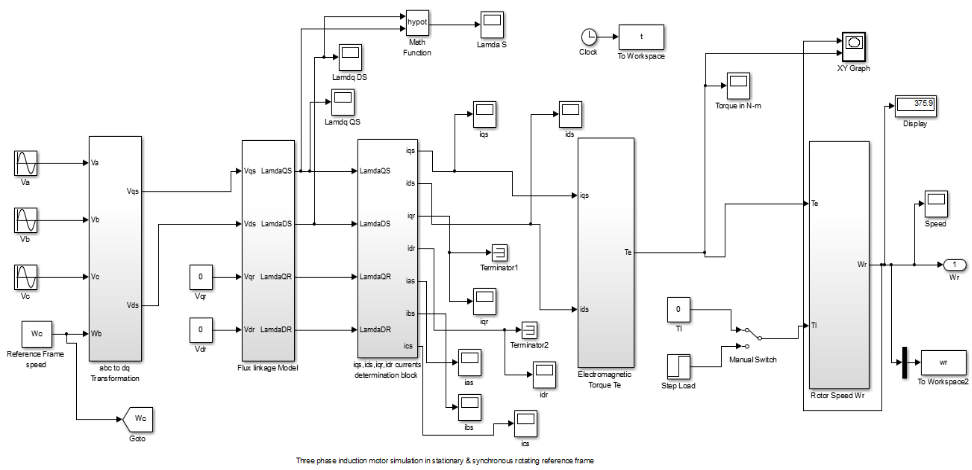


Figure 4

In this frame the speed of the reference frame is that of the stator $\omega_c = 0$

$$v_{ds} = v_m \cos(\omega_e t) \tag{19}$$

$$v_{qs} = v_m \sin(\omega_e t) \tag{20}$$

Where ω_e is base angular frequency i.e. $2\pi f$

In this frame stator voltages are displaced by 90 degrees and time varying.

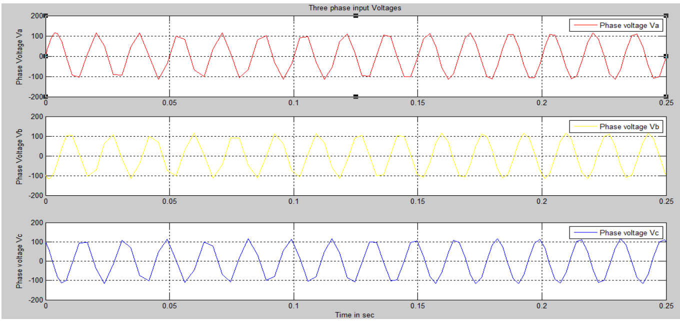


Figure 5: Input Stator V_{as} ,V_{bs} and V_{cs} Voltages

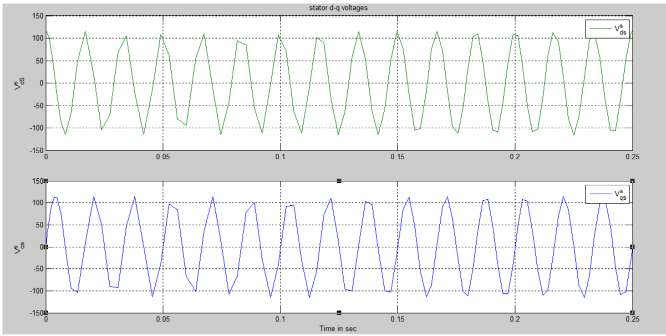


Figure 6: d-q Axis V_{ds} and V_{qs} Voltages

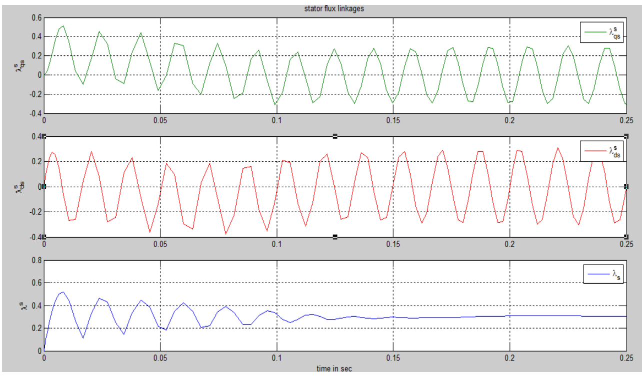


Figure 7: Stator Flux Linkages

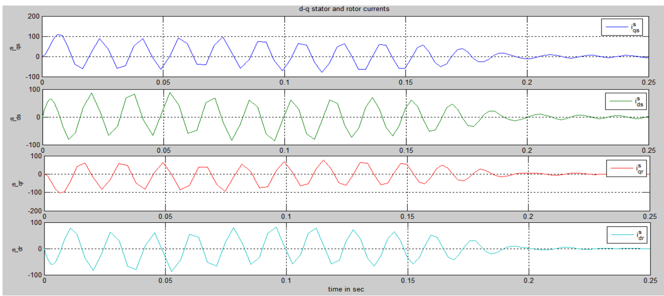


Figure 8: Stator and Rotor d-q Axis i_{ds},i_{qs},i_{dr} and i_{qr} Currents

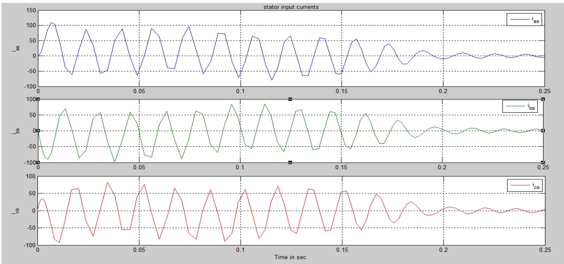


Figure 9: Stator i_{as} ,i_{bs} and i_{cs} Input Currents

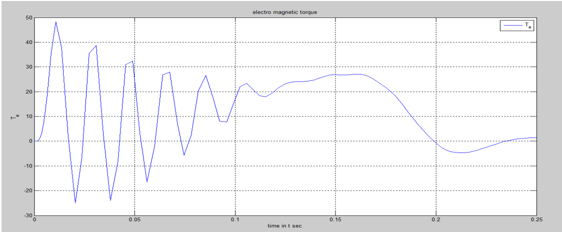


Figure 10: Electromagnetic Torque in N-m

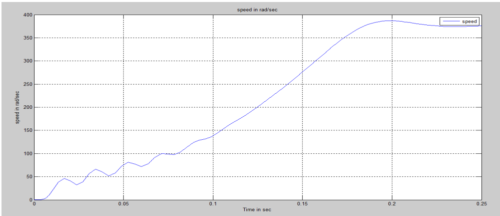


Figure 11: Speed in rad/sec

This model allows the simulation of stator controlled induction motor drives such as phase controlled and inverter controlled induction motor drives.

ROTOR REFERENCE FRAME MODEL

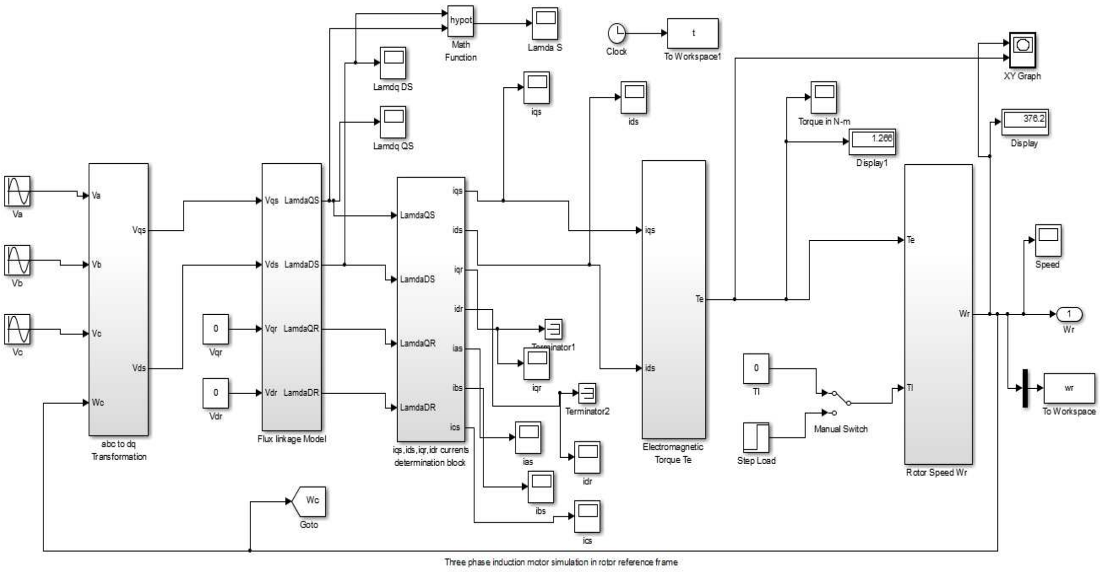


Figure 12

In this frame the speed of the reference frame is that of the rotor speed i.e. $\omega_c = \omega_r$

$$v_{ds} = v_m \cos(\omega_{sl} t) \tag{21}$$

$$v_{qs} = v_m \sin(\omega_{sl} t) \tag{22}$$

Where $\omega_{sl}$is angularslip frequency i.e. ω_e-ω_r

In this frame the stator voltages appear at slip frequency in rotor reference frame hence the currents are at slip frequency in steady state.

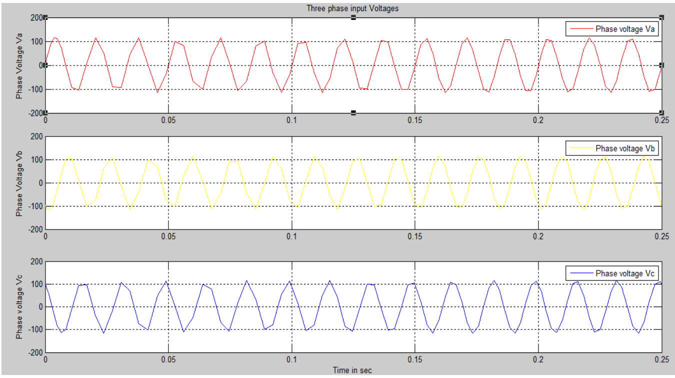


Figure 13: Input Stator V_{as} ,V_{bs} and V_{cs} Voltages

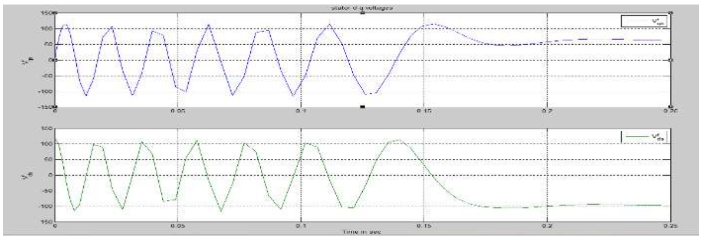


Figure 14: d-q axis V_{ds} and V_{qs} Voltages

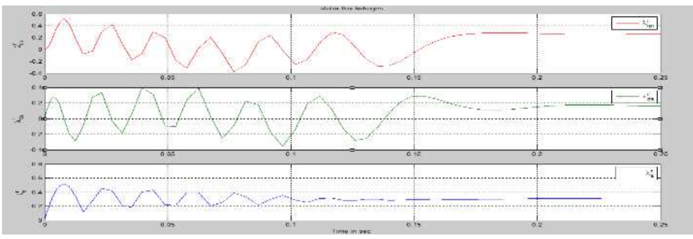


Figure 15: Statorflux Linkages

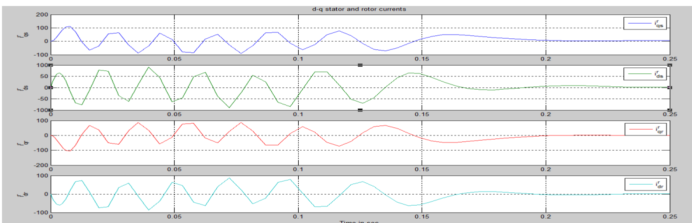


Figure 16: Stator and Rotor d-q axis i_{ds},i_{qs},i_{dr} and i_{qr} Currents

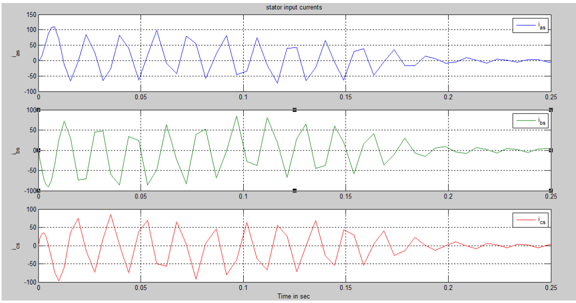


Figure 17: Stator i_{as} ,i_{bs} and i_{cs} Input Currents

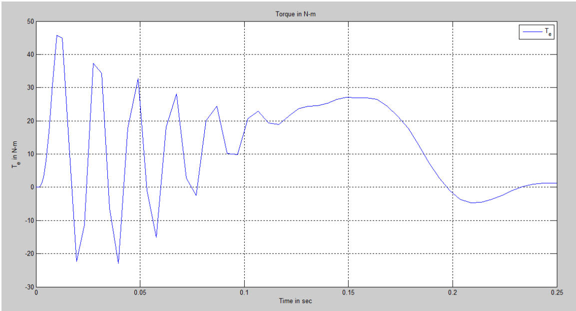


Figure 18: Electro Magnetic Torque in N-m

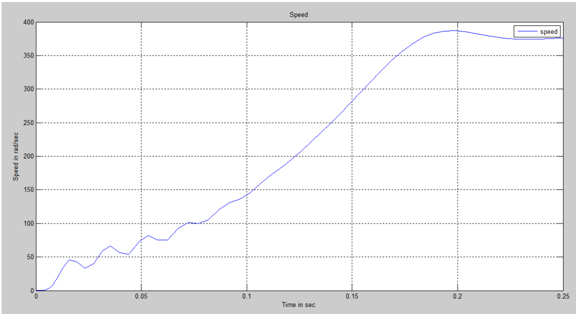


Figure 19: Speed in rad/sec

This rotating reference model is useful where the switching elements and power are controlled on the rotor side i.e. slip –power recovery scheme.

SYNCHRONOUS ROTATING REFERENCE FRAME MODEL

In this frame the speed of the reference frame is that of the rotor speed i.e. $\omega_c = \omega_e$

$$v_{ds} = v_m \tag{23}$$

$$v_{qs} = 0 \tag{24}$$

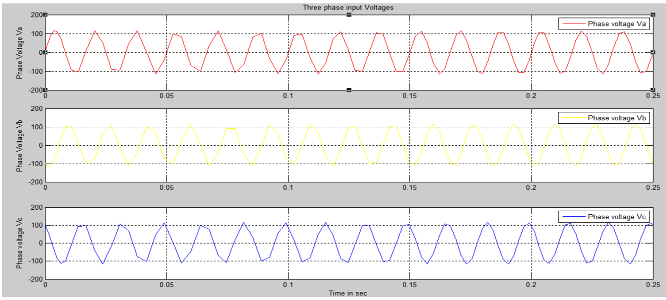


Figure 20: Input Stator V_{as} ,V_{bs} and V_{cs} Voltages

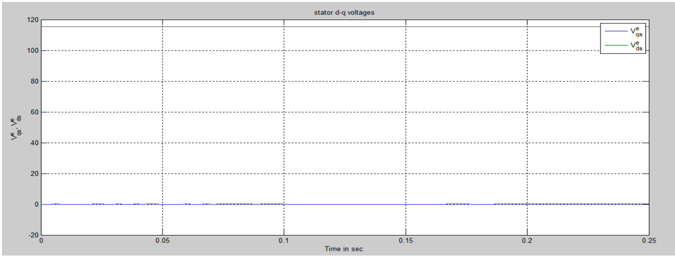


Figure 21: d-q axis V_{ds} and V_{qs} Voltages

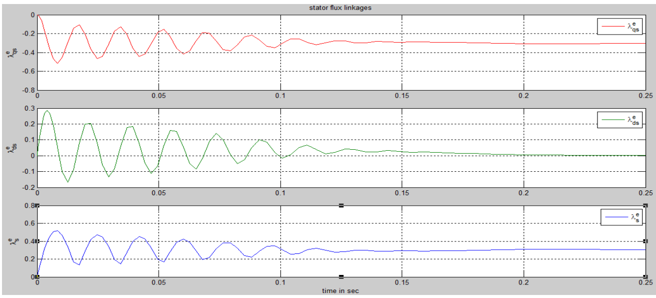


Figure 22: Stator Flux Linkages

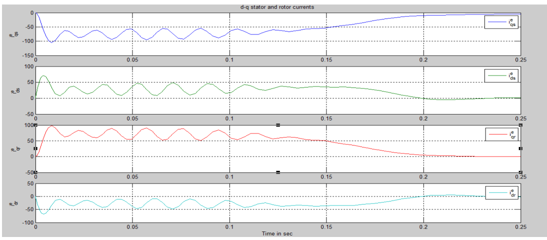


Figure 23: Stator and Rotor d-q axis i_{ds},i_{qs},i_{dr} and i_{qr} Currents

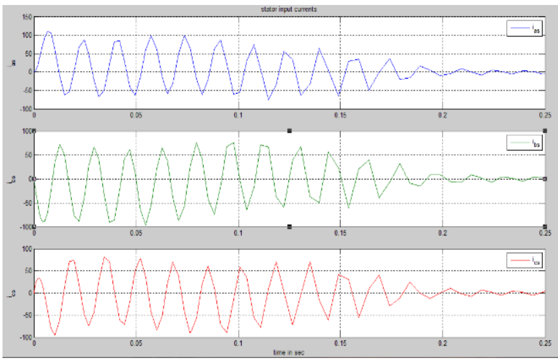


Figure 24: Statori_{as},i_{bs} and i_{cs} Input Currents

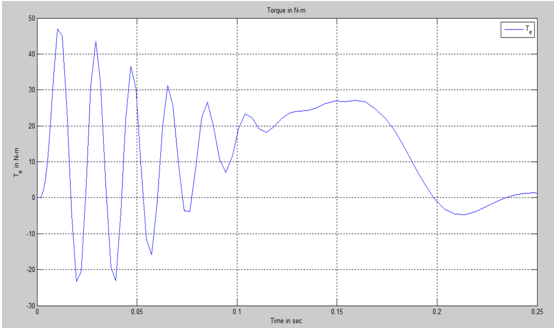


Figure 24: Electromagnetic Torque in N-m

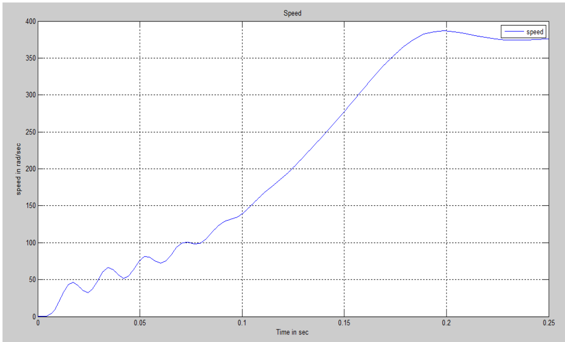
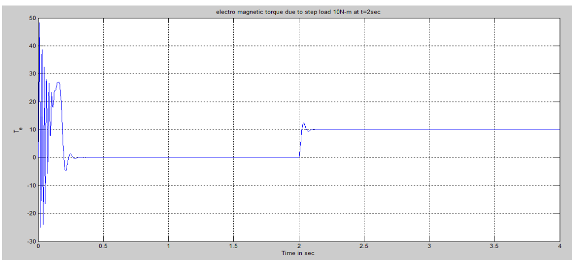


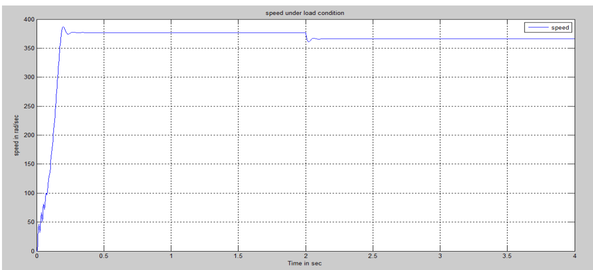
Figure 25: Speed in rad/sec

In this frame the d-q axis stator voltages are dc quantities hence the response will be dc quantities too because the system is linear. This model is useful in the development of small-signal equations of induction motor and control designing of high performance vector control induction motor drive.

INDUCTION MOTOR SIMULATION UNDER LOAD CONDITION IN STATIONARY REFERENCE FRAME

We applied load that is T_l=10N-m at t=10sec and simulated the following torque speed characteristics are observed





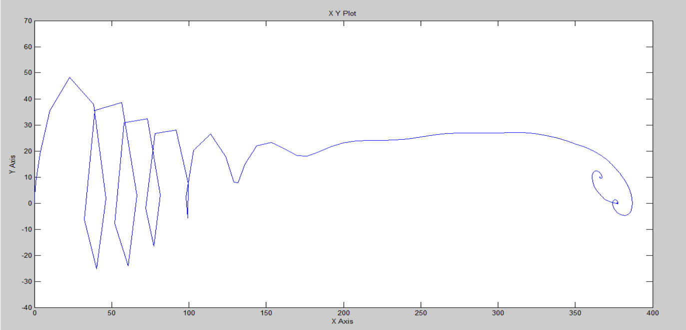


Figure 26: Torque, Speed and Torque-speed Characteristics under Load Condition

x-axis Speed in rad/sec & y-axis Torque in N-m

From above plots we observed that under direct starting of induction motor when load T_l=10N-m is applied at t =2 sec the motor is produced the electromagnetic torque T_e=10N-m at that instant and maintained same torque under steady state also to withstand the load and speed also decreased.

CONCLUSIONS

In this paper dynamic d-q axis models of squirrel cage rotor induction motor in different reference frames are presented. Using MATLAB/SIMULINK software to implement the dynamic response of squirrel cage induction in different reference frames. We observed that the machine outputs such as electromagnetic torque, speed, actual stator phase currents and magnitude of stator flux linkage **are all the same regardless of reference frames**. We observed that line start produces higher currents, flux linkages and severe torque pulsations also

In future the dynamic model is used to obtain the transient responses, small signal equations, transfer functions all of which are useful in study of converter fed induction motor drives. we will implement high performance drive control such as vector or field oriented useful for adjustable speed drive based on the dynamic d-q model of the induction machine.

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